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Investigation of Swedish krona exchange rate volatility by APARCH-Support Vector Regression

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Abstract

This paper investigates daily exchange rate volatility behaviors with a focus on a small open economy's currency, the Swedish krona (SEK), against four currencies: the U.S. dollar, Euro, the Pound Sterling (GBP), and the Norwegian krone (NOK) over the whole period from Jan. 2010 to March 2023, whereas the whole period is divided into different sub-sample periods based on the economic events. In the framework of APARCH models, we find that volatility behavior of the Swedish krona (SEK) exchange rates varies across different currency pairs (SEK being included in all cases) and sub-sample periods. Precisely, a negative asymmetric return-volatility relationship was found for the case of the SEK/EUR exchange rate, while an inverted asymmetric relationship was detected in the case of SEK/NOK exchange rate. Significant asymmetric effects of volatility in the SEK/USD and SEK/GBP exchange rates were not observed for either the whole period or the three sub-sample periods. As the return of exchange rate are all non-normally distributed, we then use a distribution-free support vector machine-based regression, called support vector regression (SVR), to estimate and forecast volatility in the framework of the chosen APARCH model for each krona exchange rate. The result shows that the SVR-APARCH based volatility forecasting performs better than the forecasting based on APARCH model estimated by maximum likelihood estimation (MLE).

JEL classification: C14, C53, F31
Keywords: Conditional volatility, volatility, SVR, Wavelet, Asymmetry, APARCH

1. Introduction

There are 'stylized characteristics' in the financial series among which are volatility clustering, the asymmetric effect, and fat-tail leptokurtosis (Kirchler & Huber, 2007; Thurner et al., 2012; Ning et al., 2015; Dzieliński et al., 2018, Baur & Dimpfl, 2019; Kim & Meddahi, 2020 among others). The change of volatility over time shows a tendency to persist: large changes in financial markets are often followed by larger changes while small changes are followed by smaller changes. This persistence of amplitudes in volatility change is termed volatility clustering. In financial markets, the asymmetric effect, in general, implies that volatility tends to be more influenced by negative returns (losses) of a corresponding size than by positive returns (gains). Furthermore, the rate of returns in financial and macroeconomic variables tend to show ‘fat tails’ or ‘outlier-prone’ irregularities. This irregularity is also known as leptokurtosis in financial literatures.

Among those volatility in the financial series, the exchange rate volatility has far-reaching implications for international trade, investment, monetary policy, and macroeconomic stability. However, within financial literature, it is not clearly supported that asymmetric volatility exists in foreign exchange markets. The primary argument suggesting the symmetric feature of exchange rate volatility is rooted in the two-sided nature of the foreign exchange market, which makes distinguishing ‘good news' from 'bad news’ challenging (Bollerslev et al., 1992). On the other hand, the presence of asymmetry in bilateral exchange rates can be explained three reasons, despite its bilateral nature. First, the asymmetry can be explained by taking the central bank’s market intervention in using foreign reserves when the currency value is falling compared to when it is increasing. As central banks intervene in one side of the market but not the other, it could result in higher volatility (McKenzie, 2002, Beine & Laurent, 2003; Galati et al., 2005; Wang & Yang, 2009). Second, the economic importance of some currencies is greater than the others. In numerous business and financial institutions, the U.S. dollar serves as the primary currency against which gains and losses are measured when compared to currencies like the Swedish Krona (SEK, hereafter). The impact known as the base-currency effect may differ based on the size and development of the local economy (Wang & Yang, 2009). Third, impact of attention and uncertainty among investors that decides asset prices. Attention as a relevant factor to asset prices were studied by Da et al. (2011), Vlastakis and Markellos, 2012; Goddard et al., 2015; Dimpfl and Jank 2016; and Dzieliński et al., (2018). Asymmetric levels of attention inherently lead to uneven levels of volatility.[1]

Unlike many previous studies that have primarily examined bilateral relationships involving major currencies, such as the U.S. dollar, this paper focuses on the daily nominal exchange rates of the currency of a 'small country,' the SEK, against four currencies among which three are major currencies: the U.S. dollar (USD), the Euro (EUR), and the British pound (GBP), and the other is a peer currency, the Norwegian Krone (NOK) over the period from January 2010 to March 2023. By a ‘small country’, it means that developments in that country do not exert any noticeable beyond the country. Consequently, in economic modeling, including exchange rate models, foreign variables are typically treated as given. Within this framework, it is reasonable to hypothesize that the currency value of small, open economies is more vulnerable to external shocks and uncertainty stemming from macroeconomic policies, political incidents, and natural disasters compared to larger economies. (Bacchetta & Chikhani, 2020).[2] During times of global

[1] Firms receiving greater overall attention tend to also encounter proportionately more attention asymmetry and subsequently exhibit increased volatility asymmetry. The association between investor attention and volatility in the foreign exchange (FX) rates of seven major currency pairs was investigated in Goddard et al. (2015). Their study presents empirical findings indicating a connection between fluctuations in investor attention and shifts in trading volume among major traders. A noteworthy observation is the clear and significant positive association between attention levels and volatility.

[2] For example, an empirical model of exchange rate volatility is introduced in Devereux and Lane (2003), where a bilateral exchange rate volatility modelling includes the level of economic developments. It is inferred that developing economies are more exposed to the external financial liabilities than the developed economies. Zhou et al. (2020) studies the impact of policy uncertainty in the Chinese exchange rate volatility.

financial uncertainty, especially the U.S. dollar, tends to strengthen while the currency values of small open economies fall (Coudert et al., 2011). On that account, this paper investigates the modeling and forecasting of exchange rate volatility, focusing on the currency value (Swedish krona, SEK) of the small-open economy, Sweden. Additionally, it explores whether there is asymmetry in volatility in its exchange rates against major currencies.

To model volatility clustering in the dataset, generalized autoregressive conditional heteroscedasticity (GARCH)-type of model, which was developed by Bollerslev (1986), has been dominantly employed so far. These models have various extensions such as exponential GARCH (Brandt & Jones, 2006; Harris et al., 2011; Epaphra, 2016), threshold ARCH (McKenzie, 2002), IGARCH (Rapach & Strauss, 2008), asymmetric power APARCH (Ding et al., 1993; Bagchi, 2016), fractionally integrated generalized autoregression conditional heteroscedastic models (FIGARCH, Vilasuso, 2002), and many more. On the other hand, the GARCH-type of models are parametric and are traditionally estimated by maximum likelihood estimation (MLE), while the efficiency of MLE will depend on if the assumption of data distribution in likelihood function implemented in MLE match to the real data distribution (Engle & González-Rivera, 1991; Bollerslev & Wooldridge, 1992; Andersen & Bollerslev, 1998). A growing attention has recently been paid to the estimation of GARCH type of models using support vector machine (SVM) based regression, also called support vector regression (SVR) which do not need priori assumption of the data distribution (Peréz-Cruz et al., 2003, Chen et al., 2010; Ou & Wang, 2010; Li, 2014; Li & Karlsson, 2023). As a unique and global solution can be found in SVR by solving a quadratic optimizing problem with a linear constraint, SVR outperforms another data driven nonparametric artificial neural network (ANN), whereas the estimation can be trapped in the local optimal solution. Chen et al. (2010) document that SVR-GARCH models show superiority in volatility forecasting than the counterparts: moving average, standard GARCH, EGARCH and ANN-GARCH models. Moreover, the SVR also has a well-performed generalization ability (Cortes & Vapnik, 1995), and will generally perform better in forecasting, compared to the model estimated by the parametric MLE method. When it comes to employing the aforementioned methods in exchange rate volatility, Chung & Zhang (2017) present the results for major currencies against the U.S. dollar showing that SVR-GARCH volatility models outperforms the existing parametric volatility models. Chen et al. (2010) uses the British Pound-US dollar daily exchange rates and show that the SVR-based GARCH model have improved predicative potential compared to the parametric GARCH based on the mean absolute error (MAE) and directional accuracy (DA).

Against this backdrop, this paper employs the Asymmetric Power ARCH (APARCH) model proposed by Ding et al. (1993) to investigate volatility clustering and asymmetric effects in the exchange rates of the Swedish Krona (SEK) against three major currencies—the U.S. dollar, the Euro, and the British Pound—and a peer currency, the Norwegian Krone (NOK).We present descriptive statistics of the dataset and conduct a series of tests to assess data distribution and the presence of volatility clustering. Subsequently, we examine the existence of asymmetric effects in the volatility of each specific dataset within the framework of APARCH models. The most suitable models, determined by the presence or absence of asymmetric effects for each specific dataset, are selected for further volatility forecasting. Finally, data-driven SVR-APARCH models are implemented for volatility forecasting and compared with results obtained from APARCH models estimated using the parametric Maximum Likelihood Estimation (MLE) method. The contributions of the paper are manifold, as outlined below:

1) Unlike most previous studies on volatility of exchange rates among the major currencies, we focus on the currency value of a small open economy, Swedish krona (SEK), against the four currencies (the U.S. dollar, the Euro, the British Pound and the Norwegian Krone).
2) A large amount of daily dataset is collected from Jan. 2010 to March 2023. We examine the volatility behaviors of the rate of return of spot exchange rates in different subperiods, which are divided based on the economic events.
3) Formal tests are conducted to detect the presence of autocorrelation in volatility, asymmetric effects, and fat-tail errors of the four return rates for the Swedish krona against the dollar, euro, pound and the krone in each subperiod, respectively. Specifically, asymmetric behaviors of the Swedish krona exchange rate volatility are examined in depth. We find that the presence of these three 'stylized characteristics' varies for each subperiod of each currency.
4) SVR is implemented for forecasting within the framework of the APARCH model and compared with results obtained from the APARCH model estimated by MLE.
5) SVR is a kernel-based regression method. When used in forecasting, the wavelet kernel is employed and compared with the commonly used Gaussian Kernel.

The paper is structured as follows. In section 2, we present the data properties, including descriptive statistics and the results of preliminary statistical test. Additionally, general descriptions of the four krona exchange rates and associated economic events over the sample period are provided in this section. Section 3 assesses the presence of asymmetric volatility effects in different subperiods for each of the exchange rate returns within the framework of APARCH models. Section 4 compares the forecasting of volatility based on the SVR-APARCH model and the APARCH model estimated by parametric MLE. The final section, section 5, includes concluding remarks.

2. Data and Statistical tests Prior to Model Comparison

2.1. **Dataset**

The data used in the paper are daily (weekdays) spot exchange rates against the Swedish krona (SEK, base currency) of four currencies (the U.S. dollar, USD; the euro, EUR; the Pound sterling, GBP and the Norwegian Krone, NOK). These data were collected from the DataStream and are averages of bid and ask prices.[3] The data span is from Jan. 1st, 2010, to Mar. 15th, 2023, resulting in a total of 3444 observations for each exchange rate (FX) series. The following Figure 1 presents the Krona exchange rates against the four currencies over the whole sample period in index values.

[3] The WM/Refinitiv Closing Spot rates are calculated by Refinitiv using data from Refinitiv Matching, EBS, Currenex, and multiple Refinitiv contributors within a five-minute window around 16:00 London time. This period represents the midpoint of the 'global day' and coincides with peak liquidity in the foreign exchange market. The daily updates are published in the Refinitiv Datastream product series by 16:45 each day. The mid rates are determined as the arithmetic mean of bid and offer prices (Source: DataStream).

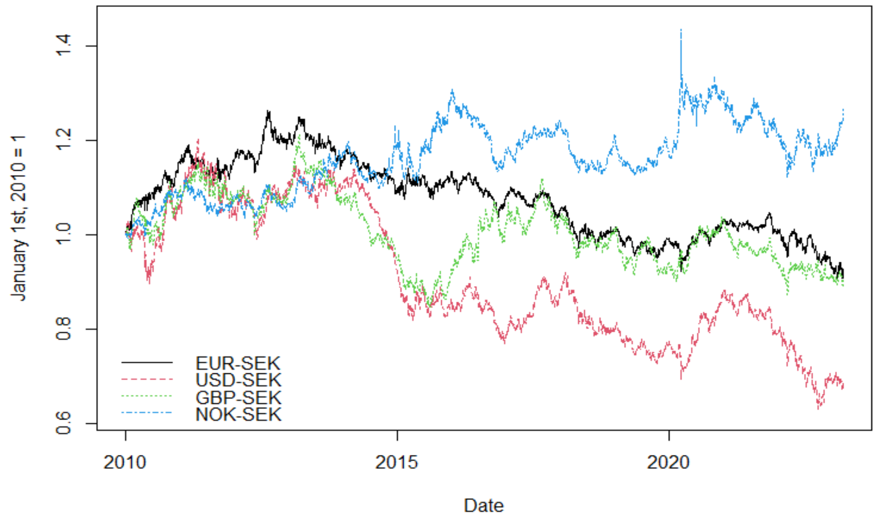

Figure 1: SEK exchange rates against the USD, Euro, Pound, and NOK, Indexed (SEK, the base currency).

Figure 1 illustrates that from 2010 until 2012, all the four exchange rates appear to follow a similar pattern, including the direction and magnitude of movements. However, this similarity in movement patterns begins to diverge around 2013. The appreciation trend of the Swedish krona against the Euro continues until 2014, while the krona's value against the other two currencies exhibits mixed movements of upswings and downswings during this period. As economic conditions in the Eurozone gradually improve and concerns over the Euros' continuity diminish, the krona's appreciation trend shifts to trending weak since 2014. The depreciation trend of the Swedish krona against the Euro continues since then. The degree of Krona depreciation against the USD and the Pound was much larger than that against the Euro during 2014-5, due to the negative interest rates adopted by the central bank of Sweden. However, the value of the krona against the Pound started to increase in 2016 following the Brexit vote, while its value against the dollar remained relatively stable during this time. Between 2018 and 2020, the four Krona exchange rates mostly moved in a similar manner. However, there have been some noticeable movements in exchange rates around the period of the Corona pandemic. At the onset of lockdowns in Europe and the U.S. in the first quarter of 2020, the Swedish krona experienced a significant depreciation against both currencies, then the currency strengthened in the second half of 2020 through 2021. Yet, the krona has clearly weakened since the beginning of 2022.[4] The SEK exchange rate against the NOK did not generally follow the same pattern during this period as observed with other krona exchange rates. Looking at the entire period and comparing it with the other krona exchange rates mentioned earlier, the SEK/NOK rate displayed somewhat different movements, which is illustrative in three respects. First, the value of SEK against NOK has been relatively stable over the period. Second, the value of the Swedish krona against the NOK demonstrates a general upward trend over the sample period, albeit with fluctuations around the trend line at shorter time intervals,

[4] The reasons for the krona's appreciation during this period are not unanimously agreed upon. Some suggest that Sweden's relative success in avoiding complete lockdown compared to other European countries and the U.S. contributed to it, while others argue that the krona's appreciation is primarily due to the change in risk sentiment as the Covid-19 crisis peaked. Additionally, unlike most G10 central banks, the central bank of Sweden (Riksbanken) did not cut rates during this period.

while SEK has been trending weaker generally against other currencies. Third, this disparity in the movement of the SEK/NOK rates is often observed in comparison to the SEK/EUR rates.

The daily krona exchange rates shown in Figure 1 are taken logarithm and denoted as p_t and, then first differencing is followed to convert these series into the daily rate of return as $r_t = 100 \times (\ln(p_t) - \ln(p_{t-1}))$, with centered return being $u_t = r_t - E_{t-1}r_t$ where $E_{t-1}r_t$ represents the expectation of the daily return based on the historical information until time t -1. For empirical study in practice, the historical average of the rate of return serves as an approximation for the expectation. The daily volatility can then be assessed by squared centered return u_t^2 (Sadeghi & Shavvalpour, 2006; Kang et al., 2009). Figure 2 shows the centered returns of the krona exchange rates against the four major currencies (u_t) and their respective volatility (u_t^2) presented side by side.

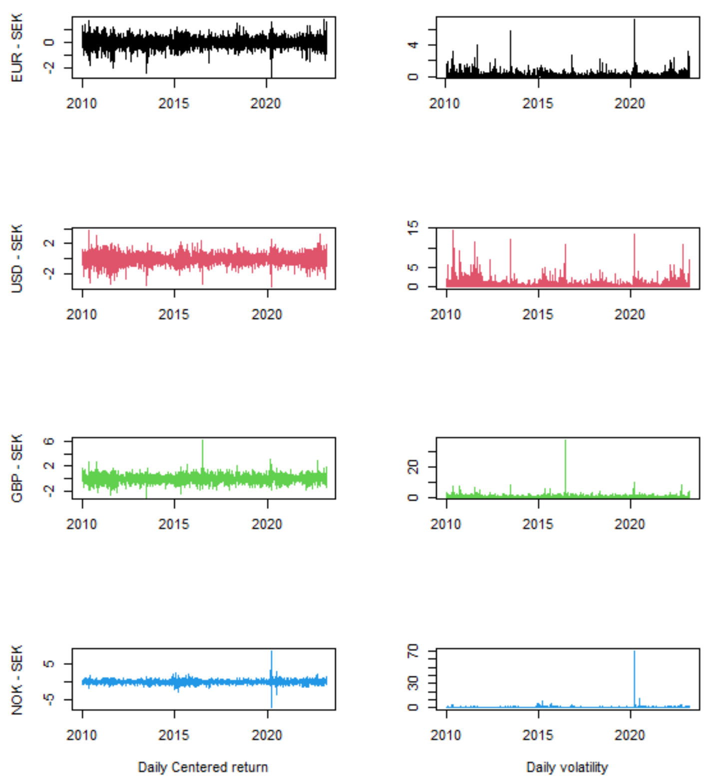

Figure 2. Daily centered exchange rate returns of the Swedish krona (on the left) and volatility (on the right).

Figure 2 shows that most of the returns are located around zero. However, it is generally observed across all exchange rates that there are some spikes in the centered return during periods of high volatility. The sample period was divided into three distinct sub-sample periods based on economic events and the associated movements in exchange rates, considering both the level and log difference series (as described earlier based on Figures 1 and 2). Additionally, this division also considered balancing the number of observations across the different sub-sample periods. The first sub-period covers the years 2010 to 2013, the second sub-period spans from the beginning of 2014 to the end of 2017, and the third sub-period includes data from 2018 until March 2023. Each of the first two sub-periods contains 1043 datasets, while the third sub-period contains 1357 datasets.

2.2 Preliminary statistical tests

To investigate the properties of the centered returns of the krona exchange rate against the four currencies, we examine descriptive statistics such as the mean, standard deviation (S.D.), skewness, and kurtosis for u_t. The normality of u_t is checked by Shapiro–Wilk test. Besides, Ljung-Box Q test is used to test the null hypothesis of no autocorrelation in volatility u_t^2 against the alternative hypothesis that there exists autocorrelation at *m* lags. We choose *m* as 1, 5 and 20, which correspond to the test of autocorrelation of 1 day, 1 week and 1 month as the dataset are daily weekday data. The descriptive statistics and test results are summarised in Tables 1-4.

Table 1. Descriptive statistics and tests for the SEK/EUR u_t and u_t^2

Statistics	Whole sample	Period 1	Period 2	Period 3
Number of obs.	3444	1043	1043	1357
Mean	9.302e-18	0.016	-0.007	-0.007
S.D.	0.393	0.449	0.337	0.387
Minimum	-1.788	-1.711	-1.204	-1.788
Maximum	2.697	2.404	1.632	2.697
Normality p value	0.000	3.454e-11	2.246e-07	1.409e-16
Skewness	0.265	0.494	0.128	0.092
Kurtosis	5.549	4.938	4.124	6.372
p value of Q(1)	6.661e-16	0.006	0.073	2.144e-12
p value of Q(5)	0.000	0.000	0.004	0.000
p value of Q(20)	0.000	3.223e-08	1.030e-11	0.000

Table 2. Descriptive statistics and tests for the SEK/USD u_t and u_t^2

Statistics	Whole sample	Period 1	Period 2	Period 3
Number of obs.	3444	1043	1043	1357
Mean	3.779e-18	0.021	-0.012	-0.008
S.D.	0.672	0.797	0.557	0.646
Minimum	-3.819	-3.819	-2.409	-3.289
Maximum	3.680	3.478	3.314	3.680
Normality p value	2.80 e-23	3.183 e-11	1.191 e-11	1.711 e-12
Skewness	0.228	0.389	-0.015	0.161
Kurtosis	5.336	4.746	5.387	5.140
p value of Q (1)	0.000	0.000	0.760	4.811e-05
p value of Q (5)	0.000	2.019e-08	0.100	0.000
p value of Q (20)	0.000	0.000	1.307e-06	0.000

Table 3. Descriptive statistics and tests for the SEK/GBP u_t and u_t^2

Statistics	Whole sample	Period 1	Period 2	Period 3
Number of obs.	3444	1043	1043	1357
Mean	-4.737e-19	0.009	0.000	-0.008
S.D.	0.574	0.631	0.582	0.518
Minimum	-6.142	-2.694	-6.142	-3.163
Maximum	2.929	2.929	2.434	1.784
Normality p value	5.125e-26	4.976e-08	6.795e-20	2.484e-14
Skewness	-0.369	0.206	-1.079	-0.337
Kurtosis	8.215	4.391	15.08	5.526
p value of Q (1)	0.006	2.850e-06	0.737	0.120
p value of Q (5)	1.851e-06	9.892e-14	0.885	0.000
p value of Q (20)	4.036e-10	0.000	0.830	4.409e-13

Table 4. Descriptive statistics and tests for the SEK/NOK u_t and u_t^2

Statistics	Whole sample	Period 1	Period 2	Period 3
Number of obs.	3444	1043	1043	1357
Mean	-3.483e-18	0.008	-0.002	-0.006
S.D.	0.503	0.380	0.478	0.597
Minimum	-6.957	-1.701	-2.849	-6.957
Maximum	8.405	1.869	2.281	8.405
Normality p value	2.144e-49	4.044e-09	1.503e-15	7.024e-41
Skewness	0.552	0.048	-0.080	0.882
Kurtosis	44.906	4.498	6.382	55.078
p value of Q (1)	0.000	0.115	3.038e-05	0.000
p value of Q (5)	0.000	0.363	4.685e-14	0.000
p value of Q (20)	0.000	8.198e-06	0.000	0.000

From the tables above, it is commonly observable that the sample means of the centred returns u_t are nearly zero, and the standard deviation are relatively high, which shows that the krona price against these currencies experience large swings in values over time, indicating high volatility. It, therefore, suggests that there exist significant risks associated with the krona value and reflects a forex market environment of instability or unpredictability due to factors such as geopolitical events, regulatory changes, or sudden shifts in investor sentiment[5] (Menkhoff, 2008 and Ur Hehman, 2013). Additionally, this feature of the centered return having near-zero mean with large standard deviations is indicative that the distribution of the centered return might be skewed or have fat tails. Tables 1-4 further confirm that u_t are not normally distributed for all the four krona exchange rates. All four cases show nonzero skewness, and the p-values for the normality tests are nearly zero.

[5] Risks involving international transactions are well outlined in Eun et al. (2024) where the four major dimensions in international finance are listed including: 1) foreign exchange rate risk; 2) Political risk; 3). Market imperfections; and 4) expanded opportunity set.

Another noticeable feature from the tables above concerns period 2. Unlike other periods (periods 1 and 3), period 2 shows no evidence of autocorrelations across different krona exchange rates at varying degrees, except for the SEK/NOK rate. In the case of the SEK/NOK, period 1 does not show autocorrelation in volatility at lags 1 and 5, while all the others show autocorrelation. At a significance level of 5%, there is no significant autocorrelation in the volatility of exchange rate returns for SEK/EUR at a lag of one day in period 2. This lack of significance is even more pronounced in the case of SEK/USD for both one-day and five-day lags. The return volatility of the SEK/GBP exchange rate shows insignificant autocorrelation across all lags (lags 1, 5, and 20 weekdays) in period 2. These results suggest that there may be a delayed effect or pattern in the returns of the SEK/EUR and SEK/USD exchange rates, which becomes apparent only after a certain period. (Melvin & Melvin, 2003; Berger et al., 2009). This pattern also suggests that, although immediate past values do not predict future returns, as indicated by the nonsignificant autocorrelation at lag 1, there may exist longer-term dependencies or patterns in the data that could potentially be exploited for forecasting or modeling purposes. While this paper does not provide detailed explanations for the observed absence of autocorrelation in volatility in the SEK/GBP exchange rate, there is one potential approach based on our understanding of market dynamics in relevant literature, the efficient market hypothesis.[6] It suggests that the rapid assimilation of information by market participants leads to all available information, including past prices and volatility, being quickly incorporated into current prices. Following this approach, the absence (or weakness) of autocorrelation suggests that forex markets were influenced by these explanatory factors of exchange rates without unexpected shocks, potentially minimizing any autocorrelation in volatility resulting from predictable patterns in economic data releases.

3. APARCH model and investigation of asymmetric volatility

To model and forecast volatility in financial series, various models can be applied, such as autoregressive conditional heteroskedasticity and stochastic volatility models. Similarly, when analyzing exchange rate volatility, it is essential to consider three 'stylized characteristics': volatility clustering, the asymmetric effect, and fat-tail leptokurtosis. The following APARCH model, a type of autoregressive conditional heteroskedasticity model, has the capability to account for all three characteristics:

$$\begin{aligned} u_t &= \eta_t \sqrt{h_t}; \quad \eta_t \sim i.i.d. D(0,1) \\ h_t^{\delta/2} &= w + \alpha(|u_{t-1}| - \gamma u_{t-1})^{\delta} + \beta h_{t-1}^{\delta/2} \end{aligned},$$

In the above model, h_t is the conditional variance of u_t with $u_t | \mathrm{I}_{t-1} \sim D(0, h_t)$, where D is the assumed distribution for u_t and I_{t-1} is available information at time $t-1$. If u_t is fat-tail distributed, D can be set as a Student-t distribution. The second equation in the APARCH model captures the relationship of h_t and u_{t-1}, and the power term δ allows this relationship to be linear or nonlinear. The coefficient γ in the APARCH model can assess the presence or absence of an asymmetrical relationship between returns and changes in volatility. If γ is significantly larger than 0, there exists an asymmetrical impact on volatility in

[6] For more detailed discussion of the efficient market hypothesis (EMH), refer to Fama (1991). Simply put, EMH means that incremental exchange rate changes are unaffected by the history of the exchange rate.

response to the positive and negative signs of u_{t-1}. Thus, under a specified distribution *D* and specific values of δ and γ, the APARCH model can account for the three 'stylized characteristics': volatility clustering, the asymmetric effect, and fat-tail leptokurtosis.

Different values of δ and γ determine different types of APARCH models. In the later section for volatility forecasting in our paper, we implement four models: (I) GARCH model with δ = 2 and γ = 0; (II) GJR model by Glosten et al. (1993) with δ = 2; (III) TS-GARCH model of Taylor (1986) and Schwert (1989) with δ = 1 and γ = 0; (IV) T-ARCH model of Zakoian (1994) with δ = 1. For the GARCH and TS-GARCH models, the γ = 0 and the sign u_{t-1} will not play any role. To identify the presence of an asymmetric effect, a formal test needs to be carried out to assess the significance of γ. Maximum likelihood estimation (MLE) is the common way to estimate the parameters $\theta = (w, \alpha, \beta, \delta, \gamma)$ in the APARCH model given the assumption of distribution *D*. Let $\hat{\theta} = (\hat{w}, \hat{\alpha}, \hat{\beta}, \hat{\delta}, \hat{\gamma})$ denote the estimated parameters, when the model is estimated, conditional volatility h_t can be predicted given u_{t-1} and h_{t-1} as $h_t^{\delta/2} = \hat{w} + \hat{\alpha}(|u_{t-1}| - \hat{\gamma}u_{t-1})^{\hat{\delta}} + \hat{\beta}h_{t-1}^{\hat{\delta}/2}$. However, for empirical datasets, the conditional volatility h_t is unobservable and needs to be approximated. Perez-Gruz et al. (2003) use $h_t^{'} = \frac{1}{5}\sum_{k=0}^{4} u_{t-k}^2$ as the approximation of h_t. Li (2014) shows that $h_t^{'} = \frac{1}{5}\sum_{k=0}^{4} u_{t-k}^2$ may lead to an over-smoothing of volatility. Therefore, this paper employs $h_t^{'} = \frac{1}{3}\sum_{k=0}^{3} u_{t-k}^2$ as an approximation for h_t.

To assess first if there is asymmetric effect in volatility of the four krona exchange rate returns across different periods, we use the corresponding dataset to estimate the asymmetric models and check the significance of the coefficient γ. Table 5 presents the p-values of tests for significance of γ for each Krona exchange rate's return across different sub-sample periods (denoted as "whole" for the entire dataset and "1", "2", and "3", respectively, for sub-periods 1, 2, 3). The tests are conducted with (a) the APARCH model, where both δ are γ set as parameters to be estimated; (b) the GJR model, where δ = 2 and γ is a parameter to be estimated; and (c) the TARCH model, where δ = 1 and γ is a parameter to be estimated. If none of the γ coefficients in the three models are significant, there is no evidence of an asymmetric effect in volatility. In this case, the GARCH and TSGARCH models will be utilized in later sections for the corresponding period when it comes to forecasting.

Table 5. p-values of test for significance of γ

P value	APARCH				GJR				TARCH			
	Whole	1	2	3	whole	1	2	3	whole	1	2	3
SEK/EUR	0.001**	0.462	< 2e-16***	0.213	0.001**	0.372	0.123	0.450	1.18e-05***	0.078	0.014*	0.000***
SEK/USD	0.270	0.633	0.648	0.234	0.377	0.912	0.736	0.268	0.108	0.267	0.645	0.105
SEK/GBP	0.809	0.445	0.577	0.547	0.392	0.441	0.117	0.634	0.953	0.469	0.578	0.905
SEK/NOK	0.000***	NA	0.504	0.037**	0.000***	0.724	0.467	0.0318*	9.23e-05***	0.901	0.002**	0.008**

Signif. codes: 0 '***'; 0.001 '**'; 0.01 '*': 0.05 '.'; 0.1 ' '; 1

There are two noticeable findings about asymmetric volatility. Fist, Table 5 shows that among the four krona exchange rate, both the SEK/EUR and SEK/NOK exchange rates exhibit asymmetric volatility in the whole sample period, as well as in periods 2 and 3, since the γ parameter is statistically significant at the 5% level in at least one of the three models. However, for the SEK/USD and SEK/GBP exchange rates, none of the γ parameters are significant at either whole period or any of the sub-period. Thus, for the SEK/USD and SEK/GBP exchange rates, the sign of the return will have no effect on volatility, indicating no asymmetric effect in volatility. Further analysis from Table 6 shows that the estimated γ values for SEK/EUR and SEK/NOK are of opposite signs. Specifically, γ values for SEK/EUR are all positive, indicating that volatility is affected more by negative returns than by positive returns, with corresponding magnitudes. In other words, volatility increases more if the SEK/EUR return was negative in a previous period compared to positive returns, which is generally observed in financial markets. Conversely, γ values for SEK/NOK are all negative, indicating that volatility is affected more by positive returns than by negative returns, with corresponding magnitudes.

Table 6. γ value of SEK/EUR and SEK/NOK exchange rate

Gamma value	APARCH				GJR				TARCH			
	whole	1	2	3	whole	1	2	3	whole	1	2	3
SEK/EUR	0.314	0.223	1	1	0.262	0.161	0.284	0.100	0.418	0.384	0.878	1
SEK/NOK	-0.293	-1	-1	-0.221	-0.284	0.082	-1	-0.206	-0.372	-0.038	-0.863	-0.320

Some reasons to this finding of a negative asymmetric return-volatility relation in the Swedish Krona exchange rate against Euro (except for period 1 from 2010 and 2013) include the base-currency effect and market sentiments, among others, as presented earlier in Introduction section. Unlike previous studies that have shown the base currency effect of the U.S. dollar, our negative asymmetric volatility in SEK/EUR exchange rate suggests that the significance of the Euro to the Swedish economy is underscored by its close ties to international trade channels within the Euro-area, as well as economic and financial integration. This would tentatively show the base-currency effect of the Euro to the Swedish Krona exchange rate. This may indicate a base-currency effect of the Euro on the Swedish Krona exchange rate. Additionally, the significance of the Euro to the Swedish economy may also explain why investors pay proportionately greater attention to the SEK/EUR exchange rate, leading to increased attention asymmetry and subsequently exhibiting increased volatility asymmetry. While our primary focus in this paper is on the existence of asymmetry in the krona exchange rates' volatility, it is important to acknowledge that there may be specific explanations for the observed asymmetry only in the krona exchange rate against the Euro.

Secondly, Table 6 shows that the estimated values of gamma for SEK/NOK are negative. This implies that an increase in the price of the Swedish krona relative to the Norwegian krone tends to result in higher volatility. Conversely, a decrease in the price of the Swedish krona relative to the Norwegian krone leads to lower volatility in the market, establishing a pattern of positive asymmetric volatility. This finding illustrates the presence of a positive asymmetric volatility-return relation in the Swedish krona exchange rate against the Norwegian krone. Similar positive asymmetric volatility effects have been reported in previous literature on commodity markets (Baur, 2012; Chiarella et al. 2016; and Chen and Wu, 2021 among others) and in cryptocurrencies (Baur and Dimpfl, 2019; Cheihk et al. 2020).[7] Not all explanations given for the other markets regarding inverted asymmetric volatility may apply to the exchange rate market for SEK and NOK. However, the finding suggests that the krona price become more volatile following unanticipated positive shocks against the Norwegian krone because investors would buy more Swedish currency than its peer currency, the Norwegian krone. While our primary focus in this paper is on the existence of asymmetry in krona exchange rates volatility, it is important to acknowledge that there may be specific explanations for the observed asymmetry in the krona exchange rate against the Norwegian krone, based on asymmetric attentions discussed in the Introduction section earlier.

After testing the statistical significance of γ values, we proceed to estimate all the four types of APARCH models for each subperiod of the four krona exchange rate returns and evaluate their performance using the criteria of mean square error (MSE) and the mean absolute deviation (MAD). The results are presented in Tables 6 and 7. The values in bold in Tables 7 and 8 correspond to the model that performs best (lowest MSE and MAD) among the four models for each subperiod.

Table 7: MSE of the four types of APARCH models

MSE	TSGARCH			GARCH			TARCH			GJR		
	1	2	3	1	2	3	1	2	3	1	2	3
SEK/EUR	0.046	0.013	0.013	0.044	0.013	0.009	0.044	0.013	0.013	**0.043**	**0.012**	**0.009**
SEK/USD	0.469	0.122	**0.268**	0.454	**0.121**	**0.268**	0.461	0.123	0.273	0.453	0.121	0.272
SEK/GBP	0.166	0.526	0.098	**0.150**	0.519	**0.093**	0.168	0.527	0.098	0.152	0.525	0.094
SEK/NOK	0.024	0.092	1.840	0.024	0.104	1.398	0.024	**0.048**	1.805	**0.021**	0.547	**1.236**

Table 8 MAD of the four types of APARCH models

MAD	TSGARCH			GARCH			TARCH			GJR		
	1	2	3	1	2	3	1	2	3	1	2	3
SEK/EUR	0.129	0.078	0.097	0.128	0.078	0.095	**0.127**	**0.078**	0.098	0.128	**0.078**	**0.095**
SEK/USD	0.403	**0.216**	**0.268**	**0.401**	0.219	0.268	0.401	0.217	0.273	0.401	0.220	0.272
SEK/GBP	**0.244**	0.247	0.189	**0.240**	0.256	**0.188**	0.245	0.247	0.189	0.240	0.253	0.188
SEK/NOK	0.104	0.163	0.231	0.106	0.165	0.219	0.104	0.162	0.227	**0.046**	**0.158**	**0.208**

[7] Baur (2012) examines the asymmetric volatility of both precious and industrial metals, particularly focusing on gold prices. The study reveals an inverted (positive) asymmetric volatility effect across all metals, with gold demonstrating a notably larger effect compared to other commodities. Chiarella et al. (2016) investigate asymmetric volatility effects in gold and oil futures and observe an inverted asymmetric effect. The explanations that were used for positive asymmetric volatility effect in commodity market (causality from inventory and spot prices to volatility and types of commodities and financialization) are not appropriate to use for this type of asymmetry in the FX market.

Tables 7 and 8 confirm the asymmetric behaviours observed in Table 5 regarding the SEK/EUR and the SEK/NOK rates. In these cases, either the asymmetric TARCH or GJR model performs best, showing the lowest MAD or MSE in the subperiods. For the SEK/USD and SEK/GBP rates, either symmetric GARCH or TSGARCH model performs best. Based on these results, in the next section, the volatility forecasting by using TARCH and GJR models for SEK/EUR and SEK/NOK are performed; while using TSGARCH and GARCH for SEK/USD and SEK/GBP. When carrying out forecasting, the data driven SVR method are used, and the forecasting results are compared with those from the MLE method.

4. Volatility Forecasting based on APARCH – SVR

4.1. Introduction SVR algorithm

Several previous studies have shown that SVR (Support Vector Regression) estimates of GARCH-type models perform better than parametric MLE, particularly in forecasting (Peréz-Cruz et al., 2003; Ou & Wang, 2010; Wang et al., 2013; Li, 2014; Li & Karlsson, 2023). SVR's objective function incorporates generalization ability, allowing for precise predictions and avoiding overfitting. Moreover, SVR is purely data-driven and does not require assumptions about data distribution, thus circumventing inefficiencies that may arise in MLE when the data distribution is misspecified. This section provides a brief introduction to SVR and its application in estimating and forecasting volatility within the framework of APARCH models.

SVR is a nonlinear regression machine learning technique and is an extension of support vector machine (SVM), which is mainly used for classification. In regression, we are given training dataset $\{(\mathrm{x}_1, y_1),...,(\mathrm{x}_N, y_N)\} \subset \Re^d \times \Re$ with $\mathrm{x}_i \in \Re^d$ being the input vector and $y_i \in \Re$ being the output scalar, i =1,…,N. Define $\varphi(\mathrm{x})$ as the nonlinear mapping function from the input space into a higher dimension feature space $\Re^k$, where $k > d$, so that linear regression in the feature space is possible. The ε-insensitive SVR intends to find a function, $f(\mathrm{x}) = \mathrm{w}^T \varphi(\mathrm{x}) + b$ with $\mathrm{w} \in \Re^k$, which achieves optimal smoothness, under the constraints that $f(\mathrm{x})$ has a maximum deviation of ε from the training data $y, i = 1,\dots, N$. The constraints of maximum deviation of ε lead to that $f(\mathrm{x})$ can capture the relationship of the input vector x and the output scalar y well, regardless of the linearity of the relationship. The requirement of smoothness corresponds to small Euclidean norm $\|\mathrm{w}\|^2$, as smaller $\|\mathrm{w}\|^2$ indicates flatter $f(\mathrm{x})$ which has a better generalization ability (Smola and Scholkopf, 1998a; 1998b). Thus, finding the solution of the function f, then, corresponds to resolving the following constrained minimization problem:

$$\text{Minimize: } \frac{1}{2}\|\mathrm{w}\|^2 .$$
$$\text{Constraints: } |y_i - f(\mathrm{x}_i)| < \varepsilon \text{ , where } \varepsilon > 0$$

To be more flexible to the outliers and avoid over-fitting, slack variables ξ_i, ξ_i^* are introduced to allow certain errors outside the ε - band. The above constraint minimization problem then becomes the following optimisation:

$$\text{Minimize } \frac{1}{2}\|\mathrm{w}\|^2 + C\sum_{i=1}^{N}(\xi_i + \xi_i^*), \text{ subject to } \begin{cases} y - \mathrm{w}^T\varphi(\mathrm{x}) - b \le \varepsilon + \xi_i \\ \mathrm{w}^T\varphi(\mathrm{x}) + b - y \le \varepsilon + \xi_i^* \\ \xi_i, \xi_i^* \ge 0 \end{cases}.$$

After introducing Lagrange multipliers α_i and α_i^*, the above constrained optimization problem ultimately becomes equivalent to minimizing the following Lagrangian objective function with respect to α_i and α_i^*:

$$L_D = \frac{1}{2}\sum_{i,j=1}^{N}(\alpha_i - \alpha_i^*)(\alpha_j - \alpha_j^*)\left\langle \varphi(\mathrm{x}_i), \varphi(\mathrm{x}_j) \right\rangle - \sum_{i=1}^{N} y_i(\alpha_i - \alpha_i^*) + \varepsilon\sum_{i=1}^{N}(\alpha_i + \alpha_i^*)$$

$$\text{subjects to } \sum_{i=1}^{N}(\alpha_i - \alpha_i^*) = 0 \text{ and } \alpha_i, \alpha_i^* \in [0, C]$$

In addition, the Karush-Kuhn-Tucker (KKT) conditions (Karush, 1939; Kuhn and Tucker, 1951) must be satisfied. The final solution of the Lagrangian objective function results in that most of the estimated α_i and α_i^* should be zero. Denote the estimated non-zero Lagrange multipliers as $\{(\hat{\alpha}_i, \hat{\alpha}_i^*)\}_{i=1}^{L}$[8], for given input vector $\mathrm{x} \in \Re^d$, the prediction of output y can then be calculated as $\hat{y} = \sum_{i=1}^{L}(\hat{\alpha}_i - \hat{\alpha}_i^*)\left\langle \varphi(\mathrm{x}_i), \varphi(\mathrm{x}) \right\rangle + \hat{b}$, where $\left\langle \varphi(\mathrm{x}_i), \varphi(\mathrm{x}) \right\rangle$ is the inner product of vectors in the feature space and $\hat{b}$ can be calculated in the training process. Thus only the few L input vectors $\{\mathrm{x}_i\}_{i=1}^{L}$ in the training sample points associated with non-zero Lagrange multipliers $\{(\hat{\alpha}_i, \hat{\alpha}_i^*)\}_{i=1}^{L}$ are used in further prediction, and they are referred to as support vectors.

Furthermore, to avoid the complexity of computing the inner product $\left\langle \varphi(\mathrm{x}_i), \varphi(\mathrm{x}) \right\rangle$ on the higher dimension in feature space, kernel function $K(\mathrm{x}_i, \mathrm{x})$ can be used to replace the inner product $\left\langle \varphi(\mathrm{x}_i), \varphi(\mathrm{x}) \right\rangle$ in $\hat{y}$ as $\hat{y} = \sum_{i=1}^{L}(\hat{\alpha}_i - \hat{\alpha}_i^*)K(\mathrm{x}_i, \mathrm{x}) + \hat{b}$ where the kernel function $K(\mathrm{x}, \mathrm{y}) = \left\langle \varphi(\mathrm{x}), \varphi(\mathrm{y}) \right\rangle$ satisfies Mercer's theorem (Mercer, 1909). The performance of the SVR is influenced by kernel selection, and the Gaussian kernel is the most commonly used kernel when there is no special prior information about the data structure (Peréz-Cruz et al., 2003; Chen & Karl, 2010; Ou & Wang, 2010).

However, when estimating and predicting the volatility in financial data, Tang et al. (2009) suggested that the wavelet kernel, as implemented in support vector machines by Zhang et (2004), can better capture volatility clustering than the Gaussian kernel. This is because the wavelet kernel is constructed on an orthonormal wavelet basis through horizontal floating and flexing, giving it more accurate localized properties such as volatility clustering. Li (2014) investigates the estimation performance and forecasting ability of SVM when the distribution of the data follows a skewed Student-t distribution, using both Gaussian and wavelet kernels. The results show that the wavelet kernel outperforms Gaussian kernel in

[8] The optimal values of Lagrange multipliers are estimated by SMO (sequential minimal optimization) method.

prediction. The current paper applies the Gaussian kernel, $K(\mathrm{x}_i, \mathrm{x}) = \exp\left(\frac{-\|\mathrm{x} - \mathrm{x}_i\|^2}{2\tau}\right)$, and then later compares it to the wavelet kernel proposed by Zhang *et al.* (2004). The C, ε and τ are hyperparameters which are predetermined empirically by cross-validation (Vapnik & Lerner, 1963; Vapnik & Chervonenkis, 1974; Vapnik, 1995). To implement SVR in APARCH framework is straight forward, as APARCH model can be viewed as one type of regression model. We define the output scalar in SVR as h_t for GARCH and GJR, and $h_t^{1/2}$ for TS-GARCH and T-ARCH model. The input vector is defined as $\mathrm{x}_t = [u_{t-1}^2,\ h_{t-1}]$ for the GARCH model, $\mathrm{x}_t = [u_{t-1}^2,\ |u_{t-1}|u_{t-1},\ h_{t-1}]$ for the GJR model, $\mathrm{x}_t = [|u_{t-1}|,\ h_{t-1}^{1/2}]$ for the TS-GARCH model and $\mathrm{x}_t = [|u_{t-1}|,\ u_{t-1},\ h_{t-1}^{1/2}]$ for the T-ARCH model. Again, for empirical dataset, the unobservable h_t can be approximated as $h_t^{'}$ mentioned in section 3.

After training $\{(\hat{\alpha}_i, \hat{\alpha}_i^*)\}_{i=1}^{L}$ for the support vectors and estimating $\hat{\theta} = (\hat{w}, \hat{\alpha}, \hat{\beta}, \hat{\delta}, \hat{\gamma})$ from MLE, we can calculate $\hat{h}_t$ based on the input vectors. If the input vectors x_t come from the in-sample data, $\hat{h}_t$ is called the estimated value for h_t. If the input vectors x_t come from the out-of-sample data, the predicted $\hat{h}_t$ is called forecasted value of h_t. The MSE and MAD calculated from the out-of-sample data can be used to evaluate the forecasting performance.

4.2. Forecasting result comparison

This section presents the results of using SVR method to forecast h_t for each subperiod of the four exchange rate returns, based on the most suitable APARCH models chosen from Section 3. Specifically, for SEK/EUR and SEK/NOK, the SVR-TARCH and SVR-GJR models are chosen, while for SEK/USD and SEK/GBP, the SVR-GARCH and SVR-TSGARCH models are chosen. To compare the forecasting results, the last 200 data points for subperiod 1 and 3, and the last 250 data points of subperiod 3, are used as out-of-sample data. The remaining in-sample data for each period is used to estimate the parameters in MLE and Lagrange multipliers in SVR. Based on the estimated models, the out-of-sample data is used to calculate the MSE and MAD. The wavelet kernel and Gaussian kernel are implemented in SVR for result comparison.

Table 9. SEK/EUR forecasting MSE and MAD

	TARCH (MSE)			GJR (MSE)			TARCH (MAD)			GJR (MAD)		
	1	2	3	1	2	3	1	2	3	1	2	3
MLE	0.075	0.008	0.041	0.072	0.009	0.039	0.147	0.057	0.128	0.148	0.055	0.126
Gaussian	0.057	0.004	0.030	0.064	0.004	0.035	0.093	0.037	0.093	0.098	0.039	0.107
Wavelet	0.056	0.004	0.029	0.056	0.004	0.031	0.090	0.037	0.090	0.089	0.037	0.091

Table 10. SEK/GBP forecasting MSE and MAD

	GARCH (MSE)			TSGARCH (MSE)			GARCH (MAD)			TSGARCH (MAD)		
	1	2	3	1	2	3	1	2	3	1	2	3
MLE	0.149	0.092	0.133	0.157	0.093	0.138	0.223	0.207	0.209	0.213	0.207	0.210
Gaussian	0.107	0.045	0.106	0.105	0.041	0101	0.138	0.130	0.148	0.134	0.115	0.142
Wavelet	0.095	0.042	0.089	0.084	0.041	0.092	0.136	0.122	0.139	0.131	0.114	0.138

Table 11. SEK/USD forecasting MSE and MAD

	GARCH (MSE)			TSGARCH (MSE)			GARCH (MAD)			TSGARCH (MAD)		
	1	2	3	1	2	3	1	2	3	1	2	3
MLE	0.301	0.059	0.498	0.300	0.061	0.509	0.303	0.171	0.452	0.289	0.180	0.452
Gaussian	0.209	0.027	0.422	0.182	0.028	0.435	0.214	0.098	0.378	0.196	0.097	0.379
Wavelet	0.172	0.026	0.474	0.194	0.027	0.391	0.184	0.094	0.385	0.188	0.095	0.363

Table 12. SEK/NOK forecasting MSE and MAD

	TARCH (MSE)			GJR (MSE)			TARCH (MAD)			GJR (MAD)		
	1	2	3	1	2	3	1	2	3	1	2	3
MLE	0.038	0.013	0.055	0.374	0.012	0.052	0.136	0.095	0.168	0.136	0.091	0.156
Gaussian	0.023	0.007	0.038	0.026	0.013	0.174	0.093	0.058	0.116	0.101	0.068	0.227
Wavelet	0.021	0.006	0.035	0.021	0.006	0.032	0.091	0.054	0.111	0.091	0.057	0.121

Tables 9 – 12 show that, in terms of forecasting, SVR generally performs much better than the MLE method, with significantly lower MSE and MAD. When using the SVR method, the wavelet kernel identifies local volatility clustering better than the Gaussian kernel, resulting in generally lower MSE and MAD. The superior performance of SVR compared to MLE is not surprising, as Tables 1-4 demonstrate that the exchange rate returns, regardless of the period or currency, are not normally distributed, leading to inefficiency problems for MLE. In contrast, the SVR method does not need any prior assumptions about data distribution for model estimation. Additionally, the SVR method has good generalization ability without overfitting the dataset and can capture nonlinear relationships, such as volatility clustering, more effectively, thus providing better forecasting in our cases. Figures 3-8 further illustrate how SVR-based forecasting compares with MLE-based forecasting for the out-of-sample dataset in each period for SEK/EUR and SEK/GBP[9]. In the figures, the solid black lines approximate the conditional variance h_t. The green dashed line represents forecasting based on the SVR method with a Gaussian kernel, the blue dashed line represents forecasting based on the SVR method with a wavelet kernel, and the red dashed line represents forecasting based on the MLE method.

[9] Here we only present the figures for the SEK/EUR and SEK/GBP to save space since the forecasting results of the SEK/NOK and SEK/USD have the same patterns as those of the other two exchange rates.

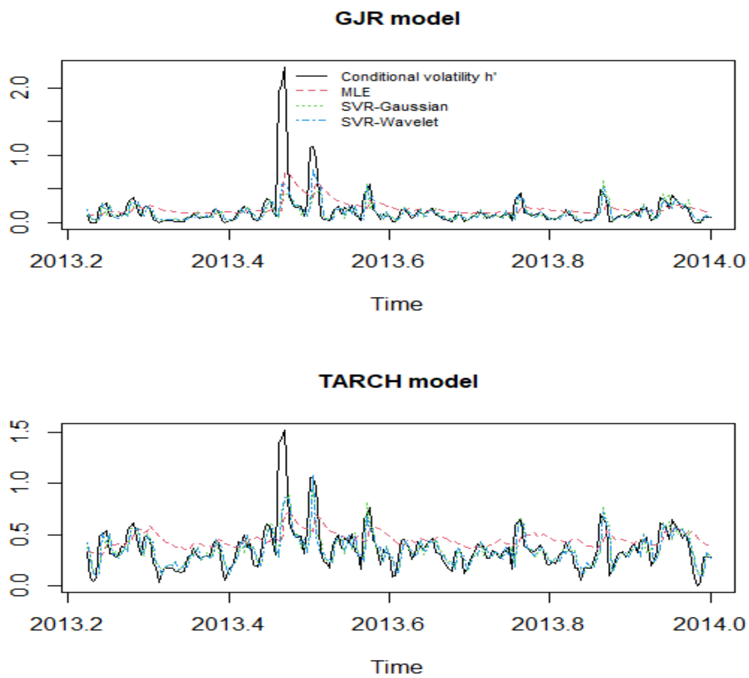

Figure 3. Forecasting for SEK/EUR, period 1

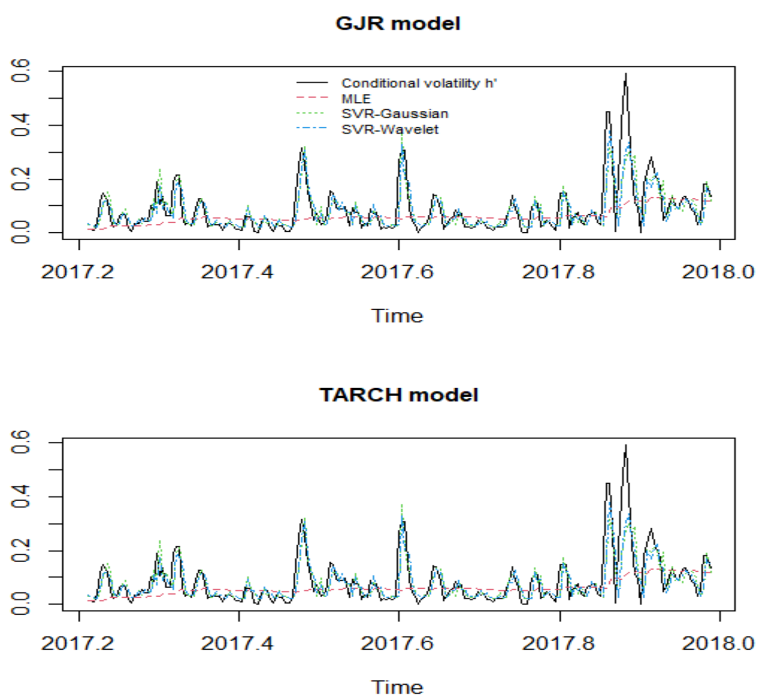

Figure 4. Forecasting for SEK/EUR, period 2

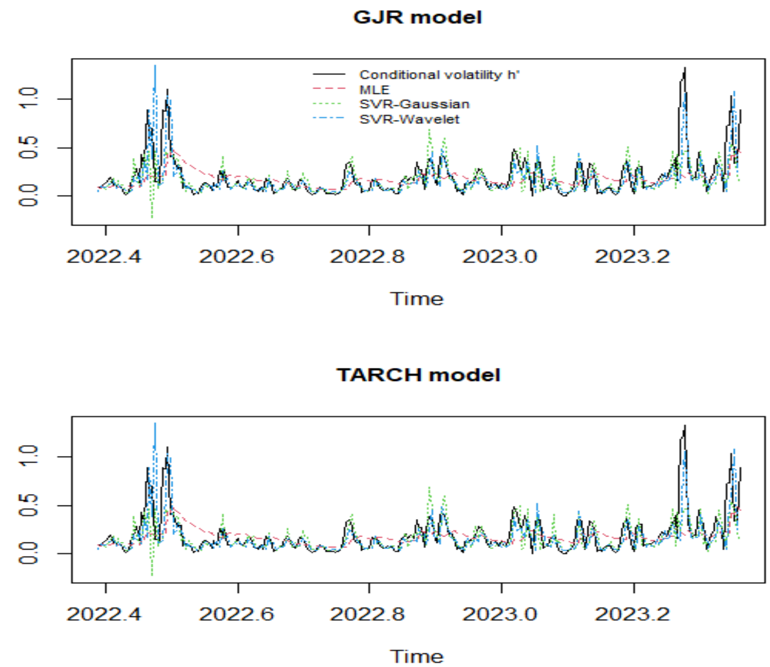

Figure 5. Forecasting for SEK/EUR, period 3

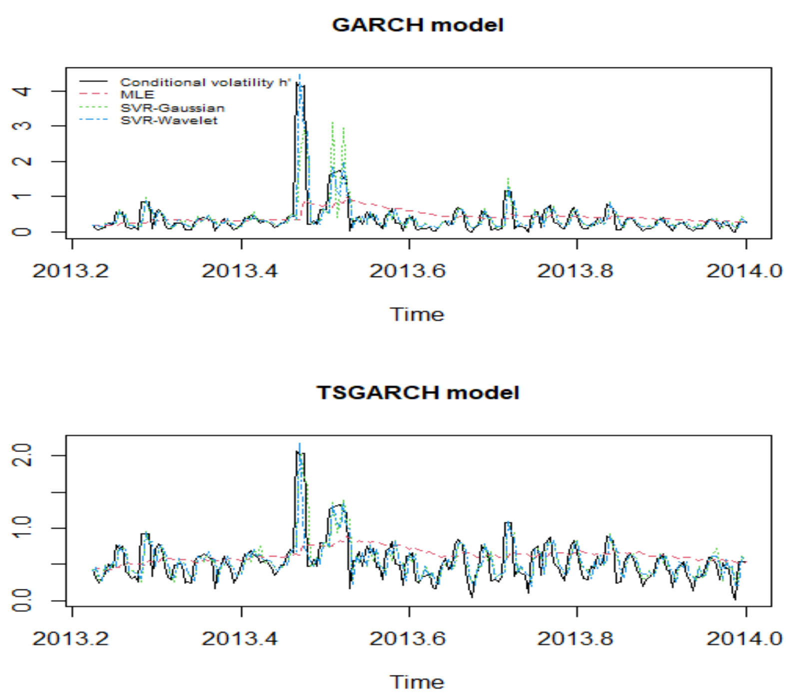

Figure 6. Forecasting for SEK/GBP period 1

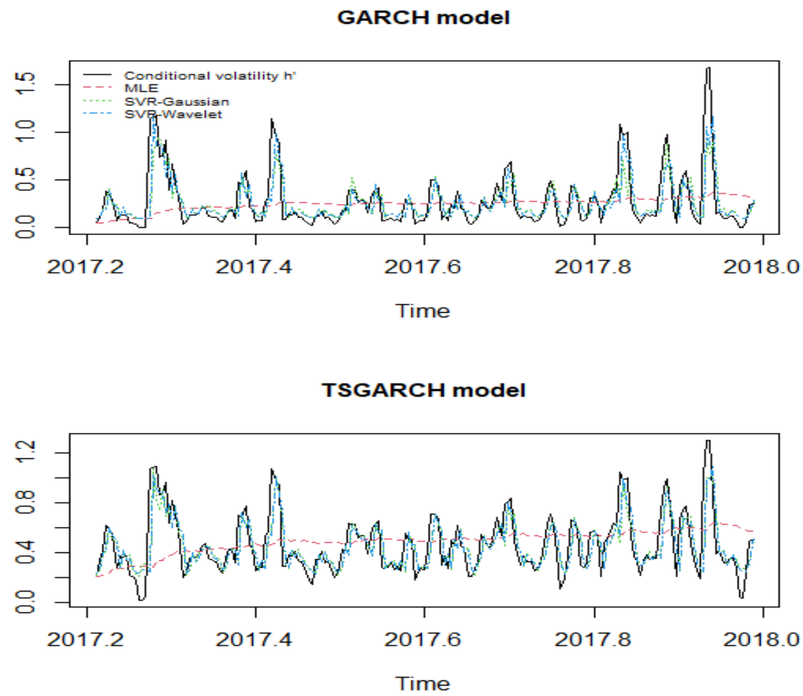

Figure 7. Forecasting for SEK/GBP period 2

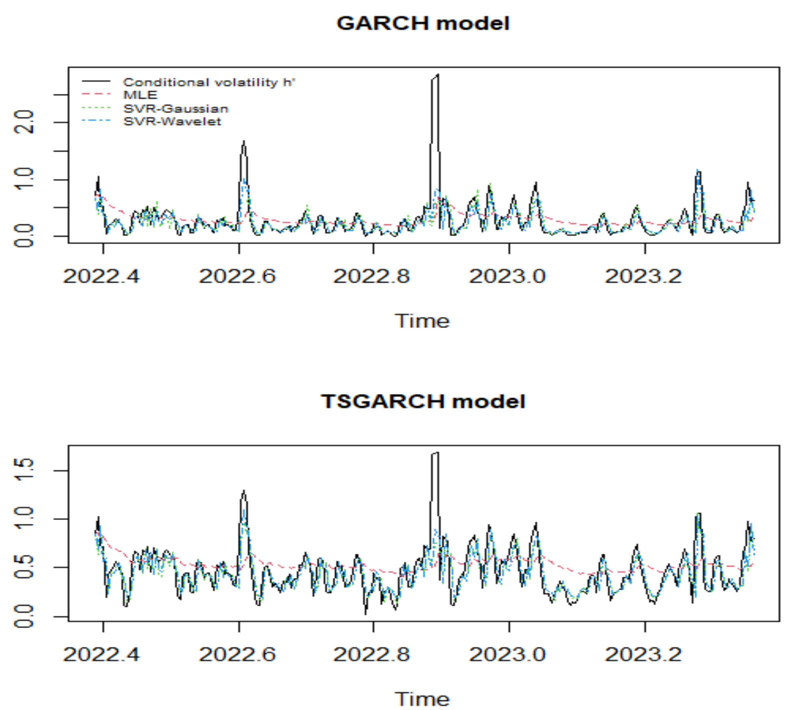

Figure 8. Forecasting for SEK/GBP period 3

We clearly observe from Figures 3-8 that the forecasting generated by MLE, although it generally captures the up and down trends as well as certain small turns in h_t, fails to capture the bumps of volatility clustering. The SVR method, whether using the Gaussian kernel or the wavelet kernel, captures the peak points much better than MLE. Additionally, the wavelet kernel produces more accurate predictions when the volatility shows wiggly peaks, due to its local identification characteristics. Another interesting finding from the figures is that, for period 2 of SEK/GBP, the forecasting generated by MLE looks like a straight line and has almost no bumps at all. This can be explained by the result found in Table 3, which shows almost no or very weak autocorrelations in u_t^2, indicating that volatility clustering is not obvious for period 2 in SEK/GBP.[10] Thus, the GARCH and TSGARCH models estimated by the parametric MLE method, which reply on the autocorrelation relationship in volatility for forecasting, failed to predict volatility in period 2.

5. Conclusion

Although volatility clustering, which is about large (small) changes in prices to be followed by large (small) changes, of either sign, is a well-known property in the financial series, we find that this is property is not always detected in the krona exchange rates that we include in our study. More precisely, we find three features about the volatility clustering. The first is that the return volatility of the Krona against the U.S. dollar and the Pound Sterling do not show clusters in the second sub-sample period (2013-2018), yet it is detected in the other two sub-periods as well as the whole sample period. The second is that return volatility of the krona against the Euro, nonetheless, shows volatility clustering for all the sub-sample periods, albeit it is weak in the sub sample period 2. The third is that return volatility of the krona against the Norwegian, also shows volatility clustering for all the sub-sample periods, yet it is much weaker in the sub sample period 1.

In other words, our findings suggest that there are no asymmetric effects in the return volatility of the SEK/USD and SEK/GBP series in all the three sub-sample periods. Asymmetries are, however, found in the two Krona exchange rates: against the Euro and the Norwegian krone. Volatility increases more when the SEK price falls against the Euro than when it appreciates (a negative return-volatility relationship), while an increase in the price of the Swedish krona relative to the Norwegian krone tends to result in higher volatility (a positive return-volatility relationship, also known as inverted asymmetric volatility). This difference in the volatility behaviour of the krona exchange rates may be explained by several factors, including market sentiment and expectations. Asymmetric attention, among other factors, can be relevant to the asymmetric volatility of the krona exchange rate against the Euro and the NOK, unlike against other major currencies in the study, namely, the U.S. dollar and the Pound. Exploring the underlying causes behind the statistical findings in relation to market participants' sentiments exceeds the scope of this paper; nonetheless, it presents avenues for future research.

Based on the properties of the exchange rate volatility, we use a distribution-free support vector machine-based regression, called support vector regression (SVR), to carry out the forecasting and compare the results with those estimated from maximum likelihood estimation (MLE). We clearly observe that SVR

[10] Different approaches that explain this absence or weak autocorrelation in return volatility are described in an earlier section, Section 2.

performs better than MLE and captures the peak points in volatility clustering much more precisely. In our study, we utilized two types of kernels in SVR. Interestingly, the wavelet kernel consistently resulted in lower Mean Squared Error (MSE) and Mean Deviation (MD) compared to the commonly used Gaussian kernel. This suggests that the wavelet kernel is a suitable choice for implementation in SVR, particularly when forecasting volatility characterized by clustering behavior. The wavelet kernel's precise localized property enhances its accuracy in capturing such patterns.

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